

# Screening of nuclear pairing in nuclear and neutron matter

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The screening potential in the  $^1S_0$  and  $^3S_1$  pairing channels in neutron and nuclear matter in different approximations is discussed. It is found that the vertex corrections to the potential are much stronger in nuclear matter than in neutron matter.

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## I. INTRODUCTION

The question of the screening of nuclear pairing is a long standing problem. In the past it has mostly been considered in neutron matter (see Ref. [1] for a list of references). However, quite recently also conjectures about the possibility of anti-screening, i.e., enhancement of pairing in finite nuclei due to surface vibrations have been put forward [2]. In Ref. [3] we also contributed to the question of the screening of the bare nucleon-nucleon pairing force in pure neutron matter and found consistently with the results of other authors that this polarization quenches quite strongly the neutron pairing in the  $S = 0$ ,  $T = 1$  channel. However, as far as we know, similar investigations have never been carried out in nuclear matter. Being interesting in its own right, our interest here is to try to make a qualitative link to finite nuclei.

Despite the fact that the validity of the local density approximation (LDA) may be questioned due to the large coherent length of the Cooper pairs, it still yields, for example, for the pair correlation energy, semi-quantitative agreement with microscopic Hartree-Fock-Bogolyubov (HFB) calculations in finite nuclei [4]. Let us therefore resume our present knowledge about nuclear pairing in different channels: as already mentioned, practically all calculations in the spin singlet channel of neutron matter indicate that polarization quenches pairing. What about  $S = 0$ ,  $T = 1$  pairing in finite nuclei? In an important very recent contribution to this subject it has been shown [5] that the bare nucleon-nucleon (NN) force yields already  $\sim 50\%$  of the measured gap in the tin isotopes. It must be mentioned that in that calculation the usual effective density dependent  $k$ -mass with  $m^*/m \sim 0.7$  at saturation has been employed. This is in line with the common lowest order approach to pairing in nuclear physics where for example with the successful Gogny D1S force [6] in HFB calculations implicitly also the  $k$ -mass is used [4]. It further should be mentioned that within the same approach and specifically with the Gogny D1S force the  $S = 0$ ,  $T = 1$  gap in symmetric nuclear matter is very close to the one obtained with the bare force for densities  $\rho \leq \rho_0/5$  whereas it drops off quite a bit slower for densities  $\rho > \rho_0/5$ . At satu-

ration ( $\rho = \rho_0$ ), the bare force yields almost negligible gap whereas D1S still yields  $\Delta \sim 0.5$  MeV (see Fig. 2 in Ref. [7]). In a LDA picture it is therefore not unreasonable that with the bare force the gap is about a factor 2 smaller than the one from this D1S-force [5]. The D1S force has therefore pairing properties which are not very far from the bare one. Remember that the weak coupling formula predicts an exponential dependence of the gap on the force (which however may be somewhat questioned in finite nuclei due to the discreteness of the spectrum). Indeed it has been shown recently that a renormalized bare force ( $v_{\text{low-k}}$ ) which is phase shift equivalent for low energies ( $\leq 300$  MeV) has shape and magnitude very similar to the Gogny D1S force [8]. The fact that D1S is close to the bare force in the  $S = 0$ ,  $T = 1$  pairing channel has also been noticed by Bertsch et al.[9] in their investigation of the neutron halo in  $^{11}\text{Li}$ .

Concluding this discussion, we can expect that medium renormalization of the bare  $S = 0$ ,  $T = 1$  pairing interaction should yield some additional attraction in nuclear matter. This is indeed what is claimed to be the case in Ref.[10] and it is also what we will find in present study of infinite nuclear matter, employing exactly the same approach as the one used before in neutron matter [3]. However, as we will see later, the effect is dramatically strong. In finite nuclei, for instance in the presently very actively studied  $N \simeq Z$  nuclei, we have the further very interesting neutron-proton ( $np$ )  $S = 1$ ,  $T = 0$  pairing channel. It is generally believed that the pairing force in this channel should be of similar strength, may be a little stronger, than the one in the  $S = 0$ ,  $T = 1$  channel [11]. In Refs.[7, 12, 13] the gap in the  $S = 1$ ,  $T = 0$  channel, calculated with the bare force (and  $k$ -mass) in infinite symmetric nuclear matter, is given as a function of density, see e.g., Fig. 1 of Ref.[13] and Fig. 1 of Ref.[7]. Because of the stronger attraction of the bare force in the deuteron channel the  $np$  gap turns out to be much larger than the neutron-neutron ( $nn$ ) or proton-proton ( $pp$ ) one. For example at maximum the  $np$  gap is about 8 MeV (!) whereas it is 2.5 MeV for the  $nn$  gap. Even at saturation  $\Delta_{np}$  still is of the order of 2 MeV (!). Clearly, the  $np$  gap in finite nuclei would turn out much too large, if the bare force was employed. Screening

therefore should give additional repulsion in the  $S = 1$ ,  $T = 0$ . We therefore have from the experimental side, and keeping in mind that LDA should at least give the right trend when going from the infinite homogeneous case to finite nuclei, definite predictions what the inclusion of polarization in a nuclear matter calculation should give, as a trend, in the  $S = 0$ ,  $T = 1$  and  $S = 1$ ,  $T = 0$  channels: additional attraction (over the bare interaction) in the former and repulsion in the latter. We will see in how far these expectations are fulfilled by the calculations.

## II. BUBBLE SCREENING

The superfluid phase of a homogenous system of fermions is characterized by the pairing field  $\Delta_k(\omega)$ , which is the solution of the generalized gap equation[14, 15, 16]

$$\Delta_k(\omega) = \sum_{k'} \int \frac{d\omega'}{2\pi i} \mathcal{V}_{k,k'}(\omega, \omega') F_{k'}(\omega'), \quad (1)$$

where  $\mathcal{V}$  is the sum of all irreducible NN interaction terms and  $F_k(\omega)$  is the anomalous propagator. Dealing with a strongly correlated Fermi system one expects the medium corrections to play a crucial role. The gap equation by itself embodies the full class of particle-particle (p-p) ladder diagrams just taking the bare interaction for  $\mathcal{V}$  [15]. The same kind of correlations are incorporated in the propagators if the self-energy is approximated by the Brueckner-Hartree-Fock (BHF) mean field [17]. From this side additional p-p corrections are unlikely to improve the predictions since the pairing rapidly vanishes at high density. On the contrary, particle-hole (p-h) correlations should play an important role at low density where the pairing gap is expected to take the largest value. But again p-h contributions have to be treated on equal footing, in the vertex corrections as well as in the self-energy in view of possible strong cancellations. This was the main concern of the study presented in Ref.[3] for  $^1S_0$   $nn$  pairing in neutron matter, further extended in a preliminary work to nuclear matter in [10]. The expansion of the interaction block  $\mathcal{V}$  and the self-energy  $\Sigma$  were both truncated to the second order in the interaction. This approximation turns out to be reasonable for the self-energy, whereas it is only indicative for the trend of the screening effects due to the interaction. In fact it was shown that the screening on the  $^1S_0$   $nn$  pairing is quite different according to whether the medium is made out of neutrons or neutron plus protons. Quantitative predictions require that the full RPA bubble series, at least, should be summed up in order to calculate the screening interaction, being aware that even this approximation could be not enough to remove the singularity associated to the low-density instability of nuclear matter.

According to the preceding discussion, we extend the study of Ref.[3] including in  $\mathcal{V}$  the full bubble series. It

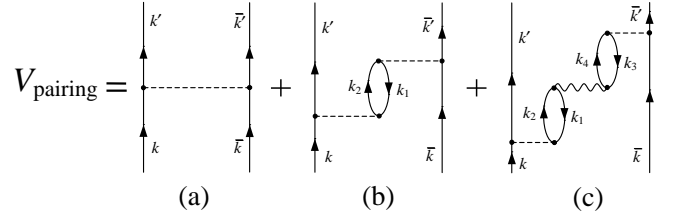


FIG. 1: Pairing interaction with screening in the RPA approximation. The dashed lines represent the Gogny interaction, the wiggly line the p-h residual interaction resummed to all orders. All vertices are to be understood as anti-symmetrised (not shown explicitly).

is convenient to develop the latter as shown in Fig. 1, where the block (c) is the correction to the results of Ref.[3]. The splitting of the bubble series into the diagrams (b) and (c) enable us to disjoin the consideration of the two external vertices connecting one particle line with one hole line from the full p-h vertex (wiggly line). So doing, the former two vertices can be calculated taking exactly into account their full momentum dependence, whereas the latter can be restricted to transitions around the Fermi momentum. In fact it is well known that the magnitude of the gap is quite sensitive to high momenta transitions, i.e. the short range part of the nuclear force [17], whereas p-h excitations are only important near the Fermi surface.

The p-h bubble series is summed up by the Bethe-Salpeter equation [14]

$$V_{S,T}^{\text{RPA}}(\mathbf{q}_1, \mathbf{q}_2, q) = V_{S,T}(\mathbf{q}_1, \mathbf{q}_2, q) + \int \frac{d^4 q_3}{(2\pi)^4} \Lambda(q_3, q) \times V_{S,T}(\mathbf{q}_1, \mathbf{q}_3, q) V_{S,T}^{\text{RPA}}(\mathbf{q}_3, \mathbf{q}_2, q). \quad (2)$$

Here  $q = (\omega, \mathbf{q})$  and  $S, T$  denote the total spin and isospin in the p-h channel.  $\Lambda(q_3, q) = -iG(q_3)G(q + q_3)$  is the polarization insertion[18]. Since, as we said, only small energy-momentum transfers significantly contribute to the pairing interaction around the Fermi energy, it is likely a valid approximation to replace the p-h residual interaction with the effective interaction  $V$  expressed in terms of Landau parameters:

$$v = f + g(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}') + f'(\boldsymbol{\tau} \cdot \boldsymbol{\tau}') + g'(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}')(\boldsymbol{\tau} \cdot \boldsymbol{\tau}'). \quad (3)$$

The remarkable advantage of this approximation is that, it makes it easy to sum up the bubble expansion of the p-h residual interaction (for a review see Ref.[19]). An additional advantage is that one is incorporating the short-range p-p correlations in the p-h effective interaction. In

terms of the Landau parameters one gets

$$N(0)V_{S,T}^{\text{RPA}}(q) = \frac{F}{1 + \Lambda(q)F} + \frac{G}{1 + \Lambda(q)G}(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}') \\ + \frac{F'}{1 + \Lambda(q)F'}(\boldsymbol{\tau} \cdot \boldsymbol{\tau}') \\ + \frac{G'}{1 + \Lambda(q)G'}(\boldsymbol{\sigma} \cdot \boldsymbol{\sigma}')(\boldsymbol{\tau} \cdot \boldsymbol{\tau}'), \quad (4)$$

where  $F = N(0)f$ ,  $G = N(0)g$ ,  $F' = N(0)f'$ ,  $G' = N(0)g'$  and  $N(0)$  is the level density on the Fermi surface.  $\Lambda(q)$  is the Lindhard function resulting from the integration of the polarization insertion [18](see Appendices). This interaction will be adopted as vertex insertion (wiggly line) in the two-bubble diagram as shown in Fig. 1c. So it represents the missing RPA correction in the one-bubble approximation for the screening, already calculated with the Gogny force [3]. Of course, this choice entails that the Landau parameters must be calculated using the same interaction, i.e. the Gogny force in our calculations, adopted for the two external interaction vertices. The Landau parameters with the Gogny force D1 [20], after RPA bubble summation at a given density, depend on energy and momentum transfer via the Lindhard functions and their asymptotic values coincide with the previous values at the same density.

At this point let us again discuss our choice of the Gogny force in the vertices (dashed lines) of Fig. 1. In principle a good approximation to the dashed lines in Fig. 1b and 1c would be a microscopic  $G$ -matrix. This is well known from many body theory. We here replace the  $G$ -matrix by the phenomenological Gogny force which has precisely been adjusted to a  $G$ -matrix calculation (see Ref.[20]). More questionable is our use of the Gogny force in the Born term of Fig. 1a. In principle in Fig. 1a the

bare force should be taken and the use of the Gogny force is for pure convenience here because a bare force scatters to very high energies. However, for better quantitative comparison of the Born terms, Fig. 1a, and the screening terms, Fig. 1b and 1c, rather the use of an effective low energy force like  $v_{\text{low-k}}$  [8], which is phase shift equivalent with the bare force at low energies, would be appropriate. Since for us such a renormalised  $v_{\text{low-k}}$  is not available in this work and because we know from [8] that  $v_{\text{low-k}}$  and Gogny forces are quite similar, we feel entitled to use the Gogny force for the Born term of Fig. 1a. In view of the fact that, as will be shown below, the contributions of the screening terms are not at all small compared with the Born term, we feel that a slight inaccuracy in the evaluation of the Born term will not at all have strong consequences for the conclusions which will be drawn in this paper. In this respect we also should mention that in the  $^1S_0$  channels, the density dependent part of the Gogny force drops out in the Born term. However, this is not the case in the  $^3S_1$  channel.

Different screening mechanisms come into play according to whether the medium is nuclear matter or neutron matter, and also whether one has pairing between like or unlike particles. Denoting by  $S$  and  $T$  total spin and total isospin in the p-p channel, the diagrams of Fig. 1 are written in the spin and isospin representation

$$\mathcal{V}_{ST} = \mathcal{V}_{ST}^{(a)} + \mathcal{V}_{ST}^{(b)} + \mathcal{V}_{ST}^{(c)}, \quad (5)$$

where

$$\mathcal{V}_{ST}^{(a)} = \sum_{\sigma_i \tau_i} \langle k \bar{k} | V_{ST} | k' \bar{k}' \rangle C_{ST},$$

$$\mathcal{V}_{ST}^{(b)} = \sum_{\sigma_i \tau_i S_i T_i k_i} (-1)^{\sigma_1 + \tau_1} C(\sigma - \sigma_1; \sigma' \sigma_2 | S_1) C(\tau - \tau_1; \tau' \tau_2 | T_1) \times C(\sigma_2 \bar{\sigma}; -\sigma_1 \bar{\sigma}' | S_2) C(\tau_2 \bar{\tau}; -\tau_1 \bar{\tau}' | T_2) \\ \times \langle k k_1 | V_{S_1 T_1} | k' k_2 \rangle \langle k_2 \bar{k} | V_{S_2 T_2} | k_1 \bar{k}' \rangle \Lambda(k_1, k_2) C_{ST}, \\ \mathcal{V}_{ST}^{(c)} = \sum_{\sigma_i \tau_i S_i T_i k_i} (-1)^{\sigma_1 + \tau_1 + \sigma_3 + \tau_3} C(\sigma - \sigma_1; \sigma' \sigma_2 | S_1) C(\tau - \tau_1; \tau' \tau_2 | T_1) \times C(\sigma_4 \bar{\sigma}; -\sigma_3 \bar{\sigma}' | S_3) C(\tau_4 \bar{\tau}; -\tau_3 \bar{\tau}' | T_3) \\ \times \langle k k_1 | V_{S_1 T_1} | k' k_2 \rangle V_{S_2 T_2}^{\text{RPA}} \langle k_4 \bar{k} | V_{S_3 T_3} | k_3 \bar{k}' \rangle \Lambda(k_1, k_2) \Lambda(k_3, k_4) C_{ST},$$

in which

$$C_{ST} = C(\sigma \bar{\sigma}; \sigma' \bar{\sigma}' | S) C(\tau \bar{\tau}; \tau' \bar{\tau}' | T)$$

and

$$C(\sigma_1 \sigma_2; \sigma_3 \sigma_4 | S) = \langle \frac{1}{2} \sigma_1 \frac{1}{2} \sigma_2 | S \sigma_1 + \sigma_2 \rangle \\ \times \langle \frac{1}{2} \sigma_3 \frac{1}{2} \sigma_4 | S \sigma_3 + \sigma_4 \rangle \delta_{\sigma_1 + \sigma_2, \sigma_3 + \sigma_4}.$$

It is worth noticing that the  $V^{\text{RPA}}$  insertion is depending only on the total spin and isospin in the p-h channel,

whereas the two external interactions are expressed in the spin and isospin coupling mixed representation of p-p and p-h channels.

Nuclear matter within the RPA treatment of p-h residual interaction suffers from a mechanical instability signaled by the well known singularity of the compression modulus at  $F_0 = -1$ . It occurs in a density domain where pairing gap is large, and its influence on the screening compromises any quantitative prediction. In neutron

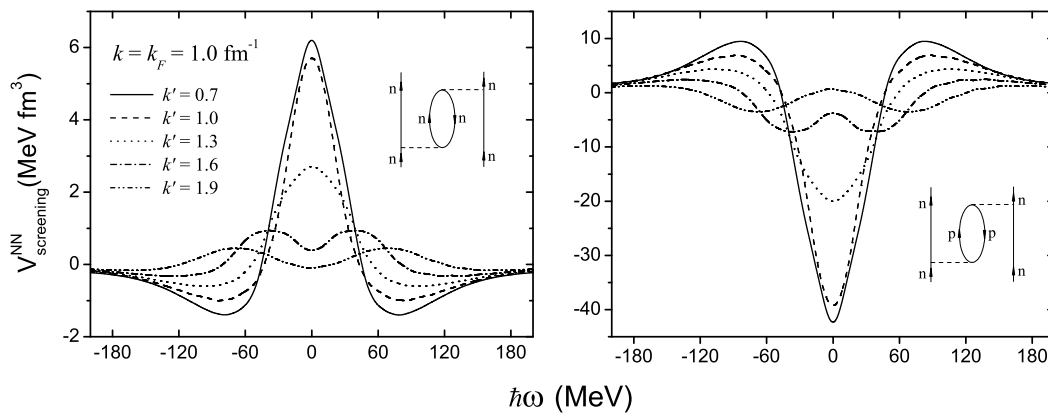


FIG. 2: Splitting of the  $^1S_0$  pairing screening potential  $\mathcal{V}_{k_F k'}(\omega)$  in nuclear matter. Left (right) panel is for the neutron (proton) p-h one-bubble insertion. The Fermi momentum is fixed to  $1.0 \text{ fm}^{-1}$ .

matter the instability would not appear, but for several interactions including Gogny D1,  $F_0$  approaches dangerously  $-1$  so making doubtful any estimate. This drawback has been cured by the Babu-Brown theory [19, 21] where the class of bubble diagrams is renormalized using the concept of induced interaction. The latter is implicitly defined as follows

$$V_{ph} = V_{dir} + V_{ind} = V_{dir} + \mathcal{V}_{\text{RPA}}(V_{ph}), \quad (6)$$

where the first term  $V_{dir}$  is the direct residual interaction ( $G$ -matrix or Gogny force in the simplest approximation [19]) and the second one is the RPA bubble series where, the vertex insertions are given by  $V_{ph}$  itself instead of  $V_{dir}$ . The construction of the induced interaction approximation (IIA) is, in general, a very complex problem [1, 19, 21, 22, 23], but it is easily realized in terms of Landau parameters, as shown in detail in the Appendix E. In principle, the propagators and the RPA calculations should be calculated in the superfluid matter. However we may guess that this consideration induces only a second order effect and we study the pairing screening only in the normal system in the present paper.

### III. RESULTS

Based on the approximations discussed in the preceding section, the screening interaction has been calculated for the  $nn$  (or  $pp$ ) isospin-triplet  $^1S_0$  channel and  $np$  isospin-singlet  $^3S_1$  channel. In the former case the screening effects due to both pure neutron matter and symmetric nuclear matter were also estimated for the sake of comparison. In the past the study of isospin-triplet pairing in nuclear matter has usually not been considered, since one mainly had in mind the problem of superfluidity in neutron stars, but for applications to nuclei the investigation of pairing in nuclear matter is also relevant.

As in a previous study [3], we used for the calculations the Gogny D1 force (see Ref. [20] and Appendix A for

details) for the external vertices, as well as for the Landau parameters describing the p-h residual interaction in the internal vertices.

#### A. Screening potential in the isospin triplet channel

Let us first consider the screening of symmetric nuclear matter and neutron matter on pairing between two nucleons in the  $^1S_0$  channel. In the one-bubble limit the neutron matter polarization gives rise to a repulsive screening of the  $^1S_0$   $nn$  channel, as shown in Fig. 3 of Ref.[3]. This is a well known result since long [23, 24, 25], and it has been interpreted as due to the dominance of the spin density fluctuations over the density fluctuations [26]. It results in a sizeable quenching of the pairing gap, reinforced by the self-energy effects [3, 27, 28]. In nuclear matter, the screening of  $nn$  pairing in the  $^1S_0$  channel can be split into two parts: neutron bubble insertion and proton bubble insertion, as shown in Fig. 2. The neutron bubble produces the same effect as for  $nn$  pairing in neutron matter, but here the additional proton polarization gives a very large attractive contribution, as shown in the right side of Fig. 2 and in Fig. 3. As a result the full medium polarization is enhancing the isospin triplet pairing (anti-screening), and produces, at least in the one-bubble approximation, a huge value of the pairing gap[10], which is cancelled only partially by the dispersive effects of the self-energy treated on the same footing.

In Fig. 2 we show the full  $\omega$  dependence of the screening part of the NN interaction for the intermediate Fermi momentum  $k_F = 1.0 \text{ fm}^{-1}$  and in Fig. 3 we show the diagonal part of the sum of the neutron and proton bubble insertions for  $k = k' = k_F$  with  $k_F$  ranging from  $0.6$  to  $1.4 \text{ fm}^{-1}$ . The total (i.e. Born plus screening on shell  $nn$  pairing interaction on the level of one bubble exchange is shown in Fig. 8 for the  $^1S_0$  channels in nuclear and neutron matter as a function of  $k_F$  (dashed lines). In

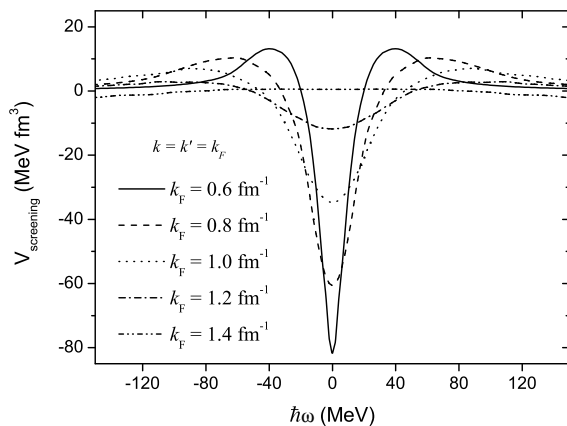


FIG. 3: Diagonal part of the sum of the neutron and proton bubble insertions in nuclear matter with  $k_F$  ranging from 0.6 to  $1.4 \text{ fm}^{-1}$ .

spite of the slight decrease of the intensity of the induced interaction at very low nuclear matter densities ( $k_F \leq 0.5 \text{ fm}^{-1}$ ), it still seems to considerably enhance the attraction of the Born term (solid line) in the low density limit. This qualitatively seems to be in line with the finding of Heiselberg et al. [29] where it was found that for a four component Fermi system (nuclear matter) the gap should be enhanced in the zero density limit, whereas, on the contrary, in a two component Fermi system (neutron matter) the gap should be reduced. This is true if the scattering lengths in all channels are approximately equal. This is not the case in nuclear matter where the  $T = 0$  scattering length is a factor three to four smaller than the  $T = 1$  one and therefore a more accurate investigation of the range  $0 \leq k_F \leq 0.5 \text{ fm}^{-1}$  is still in order but numerically very delicate.

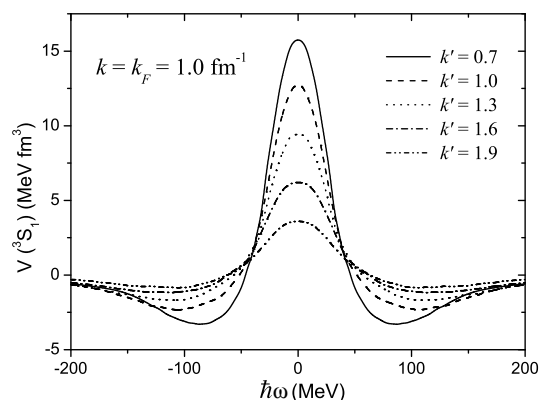


FIG. 4: Nuclear matter screening potential  $\mathcal{V}_{k_F k'}(\omega)$  in the  $^3S_1$  channel in the one bubble approximation. The Fermi momentum  $k_F$  is fixed to  $1.0 \text{ fm}^{-1}$ .

## B. Screening potential in the isospin singlet channel

In nuclear matter one also has to consider the isospin-singlet pairing, namely the pairing interaction between unlike nucleons. The dominant coupling interaction is due to the  $^3SD_1$  component of the force (in the following we neglect the  $D$  component, which makes the calculations very complicate giving however a small effect). As mentioned above, the magnitude of the gap in this case is, using the bare force, too large with respect to the values observed or predicted in finite nuclei [7, 12, 13]. Then one may expect that the screening is reducing the gap to a more physical value. And in fact it turns out to be repulsive, at least in the one-bubble approximation, as shown in Fig. 4. Actually there are energy domains where the screening potential is attractive, but it is repulsive around  $\omega = 0$ , which is most relevant for pair formation. This repulsive effect is also confirmed for the full on shell pairing force in Fig. 8 especially towards lower densities (dashed line on left panel).

## C. Screening with induced interaction

Regardless of the physical implications of the screening effects on the pairing, what we learn from the above results is that the order of magnitude of the screening or anti-screening is not small at all. This indicates that the full bubble series (RPA) should be summed up. On the other hand, we know that a pure RPA approximation, namely the bubble series with the Gogny force as residual p-h interaction, is singular in the instability region. The only way to remove this drawback is to introduce the induced interaction. So far the full RPA with induced interaction has been applied to neutron matter using a  $G$ -matrix instead of the Gogny force [1]. For the present calculations we adopted the same approximation except that, as discussed in the preceding section, the p-h residual interaction is expressed in terms of Landau parameters. This makes the resummation of the bubble series much easier to perform.

### 1. Landau parameters

According to the motivations of the preceding section, the screening interaction due to the p-h excitations is calculated approximating the residual interaction with the  $l = 0$  Landau parameters (see Ref. [30] and Appendix B). The results are plotted in Fig. 5 both for nuclear matter and neutron matter. As the Brueckner  $G$ -matrix, the Gogny force, where the short-range correlations are also incorporated, does not prevent the mechanical instability ( $F_0 < -1$ ) to appear in the low density domain of nuclear matter close up to the saturation point. Otherwise the values of the Landau parameters with the Gogny force reproduce satisfactorily the empirical values. In neutron

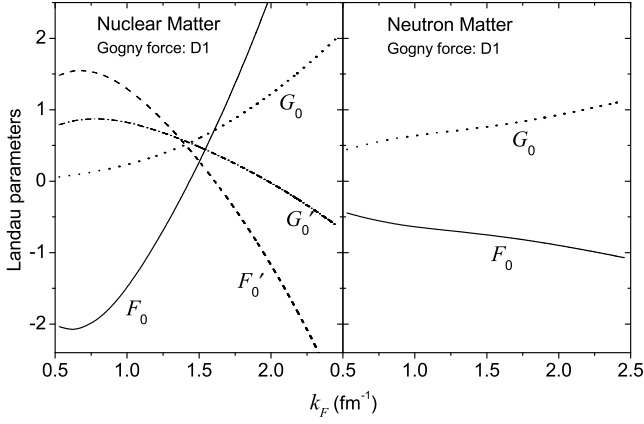


FIG. 5: Landau parameters from the Gogny force D1. Left panel is for nuclear matter and the right one for neutron matter.

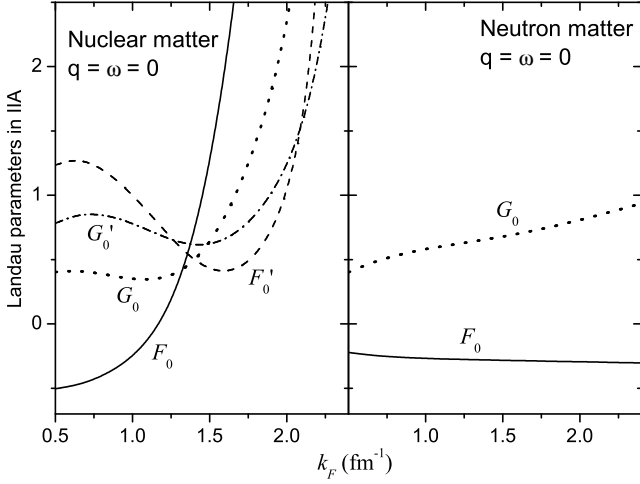


FIG. 6: The same as in Fig. 5 but in the induced interaction approximation. Only the values at the Fermi surface are depicted.

matter  $F_0$  slowly decreases, since the Gogny D1 force misses the repulsive rearrangement term in the neutron channel. The singularity due to the mechanical instability is removed by the Babu-Brown induced interaction [21] as shown by the enhancement of  $F_0$  in Fig. 6. Since the induced interaction entails a strong coupling among the components of the p-h interaction in nuclear matter (see Appendix D), the rapid rise of  $F_0$  makes the other Landau parameters to rapidly bend up also. Despite the simplicity of our approximation the induced interaction still keeps dynamical effects because of its dependence on the p-h propagator. In Fig. 7 the Landau parameters versus energy and momentum transfer in the induced interaction approximation, as shown in Appendix E, are plotted. The  $\hbar\omega = 0$  and  $q = 0$  values coincide with the static values of Fig. 6 in the induced interaction approx-

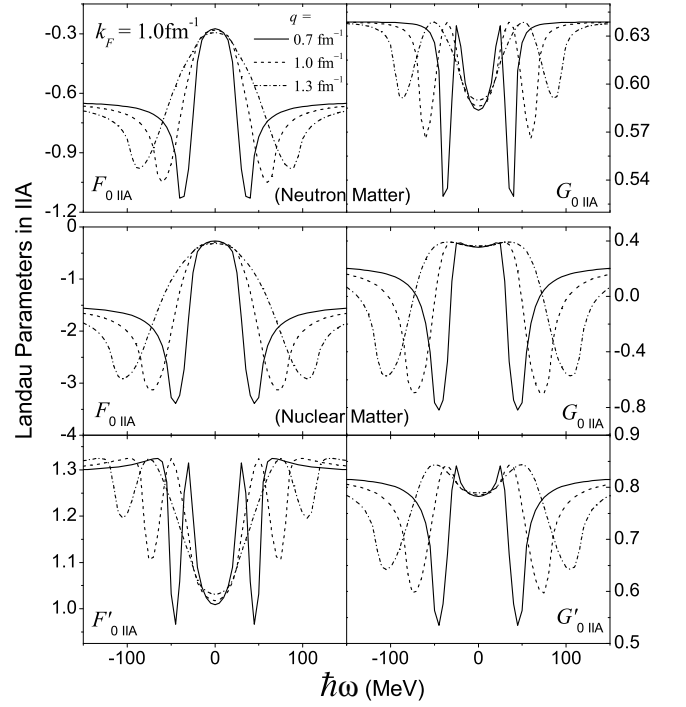


FIG. 7: Energy( $\hbar\omega$ ) and momentum-transfer( $q$ ) dependence of the induced Landau parameters (see Appendix E: E2). The upper two panels are for neutron matter and the lower four for nuclear matter. The Fermi momentum  $k_F$  is fixed to  $1.0 \text{ fm}^{-1}$ .

imation, whereas asymptotically  $|\hbar\omega| \gg 0$  they tend to the zero order limit, shown in Fig. 5. The dependence on the momentum transfer is not significant in the range shown in the figure, but in a wider range it could.

## 2. Screening potential from the full RPA residual interaction

Using the Landau parameters as vertex insertions in the p-h bubble diagrams the screening potential has been calculated in the full RPA limit with induced interaction. The pure RPA, i.e. without induced interaction, has not been considered, since it displays a singular behavior in nuclear matter. Fig. 8 summarizes the main properties of the full pairing interaction in comparison with the Gogny force. In the isospin triplet channel ( $^1S_0$ ) in neutron matter the net screening potential is repulsive because of the dominance of the repulsive spin fluctuations over the attractive density ones. A suppression is in accordance with practically all existing calculations, see e.g. [25].

For instance we also see from Fig. 3 that a substantial suppression survives at low densities. This is in agreement with the recent studies of Gori et al.[35] and Schwenk et al. [31] where also an important reduction of the gap in low density neutron matter was found. Comparing with the low density behaviour of the one-bubble

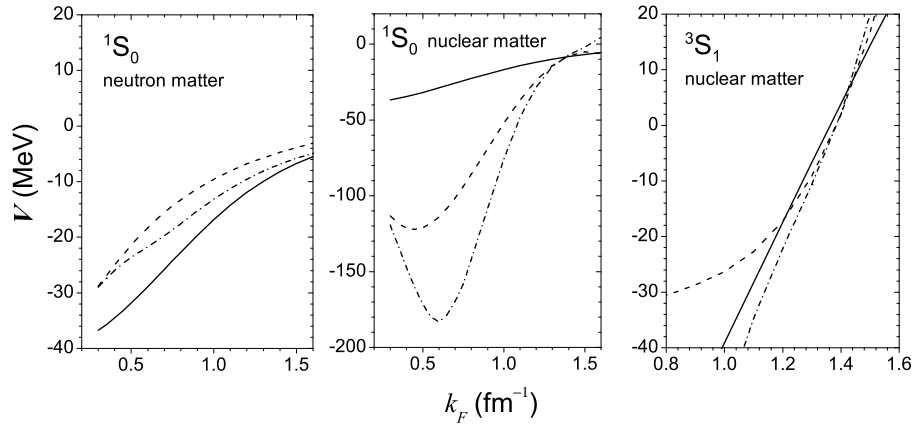


FIG. 8: Diagonal matrix elements ( $k = k' = k_F$ ) of the interaction at  $\hbar\omega = 0$ : Born term with Gogny force (solid lines), Born term with one bubble insertion (dashed lines) and full pairing potential including one and higher bubble insertion (IIA) (dash-dotted line). The baryon density corresponds to  $k_F = 1.0 \text{ fm}^{-1}$ . The Born term with the Gogny force (solid line) in the  $^1S_0$  channel is density independent, whereas it is density-dependent in the  $^3S_1$  channel.

induced interaction (see Fig.4 of Ref.[3]), we learn now that the one-bubble approximation is insufficient. At  $k_F = 0.3 \text{ fm}^{-1}$  its contribution to the pairing potential is 7.93 MeV, while the bare interaction is still -36.7 MeV. A sizeable effect is expected in the limit  $N(0)V \ll 1$ , which for the nuclear force in the  $T = 1$  channel (scattering length -18.8 fm) is fulfilled only at densities much less than  $10^{-5} \text{ fm}^{-3}$ , i.e., much beyond the range of interest of the present work.

In nuclear matter the dominance of the attractive force exerted by the proton environment on  $nn$  pairs (or neutron on  $pp$  pairs) which we already found at the one bubble level, becomes more pronounced in the full screening potential. This dominance becomes dramatic in the case of the  $^1S_0$  channel. The potential plays a role of strong anti-screening indeed. But its effect on the pairing correlations should be counterbalanced by the self-energy effect which in nuclear matter are much more pronounced than in neutron matter. The amount of compensation can be seen in Table 1. In the case of spin triplet pairing there is a sizeable compensation between self-energy and vertex corrections at any density. For the spin singlet pairing in neutron matter, as expected, both quench pair correlations, except around  $k_F = 1.0 \text{ fm}^{-1}$ . The self-energy is also reducing  $^1S_0$  pairing in nuclear matter, however, the  $T = 0$  part of the induced force is enhancing the pairing interaction very much, so that a strong (unphysical?) increase of the gap can be expected.

#### IV. DISCUSSION AND CONCLUSION

In this study we included medium polarization effects in addition to the Born term (Fig. 1a) in infinite matter in three different channels: i)  $^1S_0$  pairing in pure neutron matter, ii)  $^1S_0$  pairing in symmetric nuclear matter, iii)  $^3S_1$  pairing in symmetric nuclear matter. As already

TABLE I: Comparison of dimensionless pairing interactions in three approximations: bare interaction (first column), self-energy effects (second column), self-energy plus screening effects (third column). “np” in the second row and “nn” in the third row stand for nuclear matter and neutron matter, respectively. The factor  $Z$  is defined in Eq.(9) of Ref. [3].

channel	$k_F (\text{fm}^{-1})$	$N_0 V_{\text{bare}}$	$N_0^* Z^2 V_{\text{bare}}$	$N_0^* Z^2 (V_{\text{bare}} + V_{\text{scr.}})$
$^3S_1$	0.7	-1.21	-1.04	-1.47
	1.0	-0.95	-0.54	-0.70
	1.3	-0.21	-0.15	-0.23
$^1S_0(\text{np})$	0.7	-0.43	-0.38	-2.46
	1.0	-0.41	-0.23	-1.05
	1.3	-0.31	-0.22	-0.33
$^1S_0(\text{nn})$	0.7	-0.44	-0.33	-0.26
	1.0	-0.41	-0.33	-0.26
	1.3	-0.31	-0.26	-0.21

outlined in the introduction, one expects from the polarization contributions (self-energy + vertex) quenching of pairing in i), slight additional attraction in case ii), and quite strong quenching in iii). From Fig. 8 we can see in how much these expectation are realized within our calculations. We investigated two approximations. The first just consists in considering the one bubble exchange between two nucleons (Fig. 1b). The second also takes into account interactions between the particle and the hole (RPA). These latter interactions, are approximated by Landau parameters, renormalised by the Babu-Brown procedure (Fig. 1c).

The results in neutron matter for  $^1S_0$  pairing confirm the ones of other authors, i.e., the bare interaction is screened by 20 to 30%. This screening is stronger in the one bubble approximation than in RPA. As expected, medium polarization gives extra attraction in the  $^1S_0$

channel in symmetric nuclear matter (case ii)). However the extra attraction turns out to be enormous. With respect to the one bubble approximation the resummation of the bubbles in RPA, still enhances the effect. With such a strong total pairing force the gaps in finite nuclei would clearly be out of scale, when applying an LDA scenario. One is wondering what is happening. Maybe LDA is quantitatively completely wrong when going from nuclear matter to finite nuclei. In the past we have shown[32, 33] that for RPA, LDA is quite good for finite momentum transfers. Here the momentum transfers are concentrated around  $q = 0$  where LDA is not quite valid. This may be the reason. A second uncertainty comes from the theoretical approach. We see from Fig. 8 that going from one bubble to RPA, the effect is strong and therefore the many body approach may not be converged. Though we very carefully checked our formulas and the numerical procedure, an independent confirmation of this surprisingly large effect would be indicated. In any case it hints to the fact that also in finite nuclei anti-screening should be taken seriously as this has indeed been the case recently [2].

In the  $^3S_1$  channel of nuclear matter we expect quenching of the pairing force due to medium effects. This is indeed what is happening in the one bubble approximation. However, as in all other channels, additional bubble summation yields extra attraction, so that with RPA also in the  $^3S_1$  channel one obtains slight extra attraction.

So let us summarize the situation. At least in the one bubble approximation the trends in all three cases are as expected: repulsion for cases i) and iii), attraction for case ii). In all these cases adding RPA bubble resummation with respect to the one bubble approximation leads to considerably more attraction. In case iii) this reverses the expected trend from additional repulsion to additional (slight) attraction (with respect to the Born term). Whether these additional RPA correlations go into the right direction is not clear. Their strong effect means that the many body scheme is not converged. An approach based on a more controlled scheme like a variational theory would be in order. Also the question in how much the present study can be linked to finite nuclei needs further studies. We, however, believe that at least the trends should be the same in nuclear matter and finite nuclei.

In the end let us again comment on our use of the Gogny force in the Born term. As already mentioned this is not completely consistent since for densities larger than  $\approx \rho_0/4$  the Gogny force yields a considerably larger gap than the bare NN force (which should be used in full rigor). However, in this work we are only interested in relative trends of the Born term versus the induced force. Since the latter turns out to be a sizeable fraction of the Born term, a small change of the lowest order term will not invalidate our conclusions of this work. On the other hand we feel that the situation is not sufficiently under control to warrant the huge amount of numerical work to calculate the gap in the different channels. While in

neutron matter this may yield reasonable results which we will give in a future publication, we can already say that in the other channels and for instance for  $T = 1$  in nuclear matter the gap values will be extremely large invalidating the whole weak coupling approach of BCS.

### Acknowledgments

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### APPENDIX A: GOGNY FORCE

The appendix is devoted to develop the formalism describing the interaction and the approximations adopted in the calculations. The Gogny force, neglecting the spin-orbit term, is given by Ref. [6]

$$\begin{aligned} \langle k_1 k_2 | V | k_3 k_4 \rangle = & \sum_{i=1,2} \mu_i e^{-r_i^2 (\mathbf{k}_1 - \mathbf{k}_3)^2 / 4} (W_i + B_i P^\sigma \\ & - H_i P^\tau - M_i P^\sigma P^\tau) \\ & + \gamma (1 + P^\sigma), \end{aligned} \quad (A1)$$

where  $k \equiv (\mathbf{k}, \sigma, \tau)$  denotes the single-particle state and  $\mu, \gamma$  is defined as

$$\mu_i = (\sqrt{\pi} r_i)^3, \quad \gamma = t_3 \rho^{1/3}, \quad (A2)$$

which will be used in Appendix B, C, D. The parameters of Gogny D1 force are reported in Table I.

TABLE II: The parameters D1 for Gogny force. ( $t_3 = 1350 \text{ MeV fm}^4$ )

	$r_i(\text{fm})$	$W_i$	$B_i$	$H_i$	$M_i(\text{MeV})$
1	0.7	-402.4	-100	-496.2	-23.56
2	1.2	-21.3	-11.77	37.27	-68.81

### APPENDIX B: LANDAU PARAMETERS FROM GOGNY FORCE

The p-h residual interaction can be described in terms of the Landau parameters to be extracted from the preceding Gogny force. In order to do that we introduce the parameters

$$n_{\mathbf{k}}^{ST} = \sum_{\sigma, \tau} c_{\sigma, \tau}^{ST} n_{\mathbf{k}\sigma, \tau} \quad (B1)$$



where

$$c_{\sigma\tau}^{ST} = \begin{cases} 1 & \text{baryon density } (S=0, T=0), \\ (-1)^{\sigma-\frac{1}{2}} & \text{spin density } (S=1, T=0), \\ (-1)^{\tau-\frac{1}{2}} & \text{isospin density } (S=0, T=1), \\ (-1)^{\sigma+\tau-1} & \text{spin-isospin density } (S=1, T=1). \end{cases}$$

Spin and isospin asymmetric nuclear matter can be expressed in terms of the previous parameters as follows

$$\mathcal{H} = \mathcal{K} + \frac{1}{2} \sum_{kk'} \langle kk' | \bar{V} | kk' \rangle n'_k n_{k'}, \quad (\text{B2})$$

where  $\mathcal{K}$  is the kinetic part and  $\bar{V}$  denotes anti-symmetrized interaction. Then the Landau parameters are obtained as functional derivatives

$$F_{\mathbf{k}\mathbf{k}'}^{ST} = N(0) f_{\mathbf{k}\mathbf{k}'}^{ST} = N(0) \frac{\delta^2 \mathcal{H}}{\delta n_{\mathbf{k}}^{ST} \delta n_{\mathbf{k}'}^{ST}}. \quad (\text{B3})$$

The normalization factor  $N(0)$  is the level density and it is introduced to make the Landau parameters to be dimensionless. After expanding into Legendre polynomials, we select for our purpose only the zero-order Landau parameters. For nuclear matter we get

$$\begin{aligned} F_0 &= \sum_i [(4W_i + 2B_i - 2H_i - M_i)u_i - (W_i + 2B_i - 2H_i - 4M_i)v_i] + \frac{7}{6}\gamma, \\ F'_0 &= \sum_i [(-W_i - 2B_i)u_i - (2H_i + M_i)v_i] - \frac{3}{4}\gamma, \\ G_0 &= \sum_i [(2B_i - M_i)u_i - (W_i - 2H_i)v_i] + \frac{1}{4}\gamma, \\ G'_0 &= \sum_i [-M_i u_i - W_i v_i] - \frac{1}{4}\gamma, \end{aligned} \quad (\text{B4})$$

where  $u_i = \mu_i N(0)/4$  and  $v_i = u_i e^{-z_i} (\sinh z_i)/z_i$  and  $z_i = (r_i k_F)^2/2$  and  $|\mathbf{k}| = |\mathbf{k}'| = k_F$ ,  $k_F$  being the Fermi momentum. For nuclear matter  $N(0) = 2m^* k_F / (\pi \hbar)^2$ .  $\gamma$  and  $\mu_i$  are defined in Eq. (A2).

In the case of pure neutron matter the zero range term of the Gogny force does not contribute. Only two (zero order) Landau parameters survive, i.e.

$$\begin{aligned} F_0 &= \sum_i [(2W_i + B_i - 2H_i - M_i)u_i - (W_i + 2B_i - H_i - 2M_i)v_i], \\ G_0 &= \sum_i [(B_i - M_i)u_i - (W_i - H_i)v_i], \end{aligned} \quad (\text{B5})$$

where now  $u_i = \mu_i N(0)/2$  and  $N(0) = m^* k_F / (\pi \hbar)^2$  is the level density of neutron matter,  $k_F$  being the neutron-matter Fermi momentum.

## APPENDIX C: ONE-BUBBLE SCREENING INTERACTION

The medium polarization effects on the NN interaction in the RPA limit are given by the bubble series. The one-bubble term (diagram) has the following expression,

$$V^{(2)} = - \sum_{k_1 k_2} \langle k k_1 | \bar{V} | k' k_2 \rangle \langle k_2 \bar{k} | \bar{V} | k_1 \bar{k}' \rangle L_{k_1 k_2}(\omega), \quad (\text{C1})$$

which is suitable for the pairing interaction where one particle in the state  $k \equiv (\mathbf{k}, \sigma, \tau)$  couples to one particle in the state  $\bar{k} \equiv (-\mathbf{k}, \sigma', \tau')$ . The function  $L$  is the polarization part [18]

$$L_{k_1 k_2}(\omega) = \int \frac{d\omega'}{2\pi i} \tilde{G}_{k_1}(\omega') G_{k_2}(\omega - \omega'). \quad (\text{C2})$$

After  $\omega$ -integration within the single-pole approximation for the Green's function, one obtains

$$L_{k_1 k_2}(\omega) = \frac{n_{k_2}(1 - n_{k_1})Z_{k_2}}{\omega_{k_1} + \omega_{k_2} + \omega - i\eta} + \frac{n_{k_1}(1 - n_{k_2})Z_{k_1}}{\omega_{k_1} + \omega_{k_2} - \omega - i\eta},$$

where the  $Z$ -factors embody the self-energy effects. In the coupled case, Eq.(C1) becomes,

$$V_{\text{ST}}^{(2)}(k, k', \omega) = -\frac{1}{4\pi} \int \sum_{\mathbf{k}_1 \mathbf{k}_2, ij} V_{ij}^{(2)}(\text{ST}) L_{k_1 k_2}(\omega) d\Omega_{\mathbf{k}} d\Omega_{\mathbf{k}'}, \quad (\text{C3})$$

where ST stands for  $^1S_0$  or  $^3S_1$  channel, and  $i, j$  takes 1 and 2. In the neutron matter medium,

$$\begin{aligned} &V_{ij}^{(2)}(^1S_0) \\ &= 2(\alpha_i - \beta_i)(B_i - H_i - M_i + W_i)[\beta_j(H_j - W_j) \\ &\quad + \eta_j(B_j - M_j)] + [\alpha_i(H_i - W_i) + \beta_i(B_i - M_i)] \\ &\quad \times [\beta_j(M_j - B_j) + \eta_j(W_j - H_j)], \end{aligned} \quad (\text{C4})$$

where  $\alpha_i = \mu_i \exp[-\frac{1}{4}r_i^2(\mathbf{k} - \mathbf{k}_2)^2]$ ,  $\beta_i = \mu_i \exp[-\frac{1}{4}r_i^2(\mathbf{k} - \mathbf{k}')^2]$ ,  $\eta_i = \mu_i \exp[-\frac{1}{4}r_i^2(\mathbf{k} - \mathbf{k}_1)^2]$ , and in nuclear matter medium,

$$\begin{aligned} &V_{ij}^{(2)}(^1S_0) \\ &= -(\alpha_i H_i + \beta_i B_i + \frac{1}{2}\gamma)(\beta_j B_j + \eta_j H_j + \frac{1}{2}\gamma) \\ &\quad + 2(\alpha_i - \beta_i)(B_i - H_i - M_i + W_i)[\beta_j(H_j - W_j) \\ &\quad + \eta_j(B_j - M_j)] + 2[\alpha_i(H_i + M_i) + \beta_i(B_i + W_i) \\ &\quad + \gamma][\beta_j W_j + \eta_j M_j + \frac{1}{2}\gamma] - [\alpha_i(W_i - H_i) \\ &\quad + \beta_i(M_i - B_i)][\beta_j(M_j - B_j) + \eta_j(W_j - H_j)]. \end{aligned} \quad (\text{C5})$$

In  $^3S_1$  channel,

$$\begin{aligned} &V_{ij}^{(2)}(^3S_1) \\ &= -(\alpha_i B_i + \beta_i H_i + \frac{1}{2}\gamma)(\beta_j H_j + \eta_j B_j + \frac{1}{2}\gamma) \\ &\quad + 2[\alpha_i(M_i - B_i) + \beta_i(W_i - H_i)][\beta_j W_j + \eta_j M_j \\ &\quad + \frac{1}{2}\gamma] + 2(\alpha_i - \beta_i)(M_i - W_i + H_i - B_i)[\beta_j(B_j \\ &\quad + W_j) + \eta_j(H_j + M_j) + \gamma] - [\alpha_i(B_i + W_i) + \beta_i(H_i \\ &\quad + M_i) + \gamma][\beta_j(H_j + M_j) + \eta_j(B_j + W_j) + \gamma], \end{aligned} \quad (\text{C6})$$

where  $\gamma$  is density dependent, as defined in Eq.(A2).

## APPENDIX D: RPA SCREENING INTERACTION

The summation of the full RPA bubble series can be carried out only in few cases [34], as explained in Sec. II. A reasonable approximation which preserves the correct evaluation of vertex insertions between p-p external lines and the p-h internal lines is depicted in Fig. 1(c). It is based on a two-bubble approximation where the internal vertex is replaced by the full RPA p-h interaction

$$V^{(3)} = \sum_{k_1 k_2 k_3 k_4} \langle k k_1 | \bar{V} | k' k_2 \rangle L_{\mathbf{k}_1 \mathbf{k}_2}(\omega) \times \langle k_2 k_3 | \tilde{V}_{\text{RPA}} | k_1 k_4 \rangle L_{\mathbf{k}_3 \mathbf{k}_4}(\omega) \langle k_4 \bar{k} | \bar{V} | k_3 \bar{k}' \rangle. \quad (\text{D1})$$

The momentum transfer in the pairing interaction is  $\mathbf{q} \equiv \mathbf{k} - \mathbf{k}' = \mathbf{k}_2 - \mathbf{k}_1 = \mathbf{k}_4 - \mathbf{k}_3$ . Using the Landau parameters,  $\tilde{V}_{\text{RPA}}$  is only depending on  $\mathbf{q}$ . The latter is calculated using the Landau parameters for which the RPA summation can be easily performed. In turn, this is a reasonable approximation, since the relevant p-h excitations are those nearby the Fermi surface where the Landau parameters are calculated. Therefore, in the Landau approximation,

$$\tilde{V}_{\text{RPA}} = \tilde{f} + \tilde{g} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \tilde{f}' \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \tilde{g}' (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2), \quad (\text{D2})$$

where  $\tilde{f}$ ,  $\tilde{g}$ ,  $\tilde{f}'$ ,  $\tilde{g}'$ , including so called induced interaction, will be given in Appendix E. In the coupled case, the interaction with full RPA (the bubble series summation with two and higher bubbles) is

$$V_{ST}^{(3)} = \frac{1}{4\pi} \int d\Omega_k d\Omega_{k'} \sum_{\mathbf{k}_2 \mathbf{k}_4, ij} V_{ij}^{(3)}(ST) \quad (\text{D3})$$

where  $ST$  could be  $^1S_0$  or  $^3S_1$ . For  $^1S_0$  channel,  $V_{ij}$  takes the form

$$V_{ij}^{(3)} = (P_i \alpha_i + Q_i \beta_i)(P_j \alpha_j + Q_j \eta_j) \tilde{f} - 3(X_i \alpha_i + Y_i \beta_i)(X_j \alpha_j + Y_j \eta_j) \tilde{g} \quad (\text{D4})$$

in neutron matter, where  $\alpha_i = \mu_i \exp[-r_i^2(\mathbf{k} - \mathbf{k}')^2/4]$ ,  $\beta_i = \mu_i \exp[-r_i^2(\mathbf{k} - \mathbf{k}_2)^2/4]$ ,  $\eta_i = \mu_i \exp[-r_i^2(\mathbf{k} + \mathbf{k}_4)^2/4]$ ,  $P_i = B_i - 2H_i - M_i + 2W_i$ ,  $Q_i = -2B_i + H_i + 2M_i - W_i$ ,  $X_i = B_i - M_i$ ,  $Y_i = H_i - W_i$ , and

$$V_{ij}^{(3)} = (A_i \alpha_i - E_i \beta_i + \frac{3}{2}\gamma)(A_j \alpha_j - E_j \eta_j + \frac{3}{2}\gamma)(\tilde{f} - \tilde{f}') - 3(C_i \alpha_i + D_i \beta_i + \frac{1}{2}\gamma)(C_j \alpha_j + D_j \eta_j + \frac{1}{2}\gamma)(\tilde{g} - \tilde{g}') \quad (\text{D5})$$

in nuclear matter where  $A_i = 2B_i - 2H_i - M_i + 4W_i$ ,  $E_i = 2B_i - 2H_i - 4M_i + W_i$ ,  $C_i = 2B_i - M_i$ ,  $D_i = 2H_i - W_i$ . For  $^3S_1$  channel, we have

$$V_{ij}^{(3)} = (A_i \alpha_i - E_i \beta_i + \frac{3}{2}\gamma)(A_j \alpha_j - E_j \eta_j + \frac{3}{2}\gamma)(\tilde{f} - \tilde{f}') + (C_i \alpha_i + D_i \beta_i + \frac{1}{2}\gamma)(C_j \alpha_j + D_j \eta_j + \frac{1}{2}\gamma)(\tilde{g} - \tilde{g}'). \quad (\text{D6})$$

## APPENDIX E: INDUCED INTERACTION

The induced interaction is given by Eq.(6), i.e.

$$\tilde{V}_{\text{RPA}} = V_{\text{dir}} + \mathcal{V}_{\text{ind}}. \quad (\text{E1})$$

where the induced interaction  $\mathcal{V}_{\text{ind}}$  is the bubble series with the full interaction itself inserted in the interaction vertices. Its explicit expression, in nuclear matter case, is,

$$\begin{aligned} \tilde{f} &= f_d + f_{\text{ind}}, & \tilde{f}' &= f'_d + f'_{\text{ind}}, \\ \tilde{g} &= g_d + g_{\text{ind}}, & \tilde{g}' &= g'_d + g'_{\text{ind}}, \end{aligned} \quad (\text{E2})$$

where

$$\begin{aligned} 4f_{\text{ind}} &= A_f + 3A_g + 3A_{f'} + 9A_{g'}, \\ 4g_{\text{ind}} &= A_f - A_g + 3A_{f'} - 3A_{g'}, \\ 4f'_{\text{ind}} &= A_f + 3A_g - A_{f'} - 3A_{g'}, \\ 4g'_{\text{ind}} &= A_f - A_g - A_{f'} + A_{g'}, \end{aligned} \quad (\text{E3})$$

and

$$\begin{aligned} A_f &= \frac{\Lambda}{1 + N(0)\tilde{f}\Lambda} \tilde{f}^2, & A_{f'} &= \frac{\Lambda}{1 + N(0)\tilde{f}'\Lambda} \tilde{f}'^2, \\ A_g &= \frac{\Lambda(q)}{1 + N(0)\tilde{g}\Lambda} \tilde{g}^2, & A_{g'} &= \frac{\Lambda(q)}{1 + N(0)\tilde{g}'\Lambda} \tilde{g}'^2. \end{aligned} \quad (\text{E4})$$

While in neutron matter case, only  $\tilde{f}$  and  $\tilde{g}$  is remained in Eq.(E2) and the self-contained equation is

$$\begin{aligned} 2f_{\text{ind}} &= A_f + 3A_g, \\ 2g_{\text{ind}} &= A_f - A_g, \end{aligned} \quad (\text{E5})$$

where  $A_f$  and  $A_g$  has the same form as in Eq.(E3). Usually Eq.(E2) and (E5) is solved iteratively, in which the first guess is the direct term  $f_d$ . The simplest approximation to calculate the induced interaction is given expressing  $f$  in terms the  $L = 0$  Landau parameters where  $\Lambda = \Lambda(q, 0)$  is the Lindhard function in the static limit  $\omega = 0$ .

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